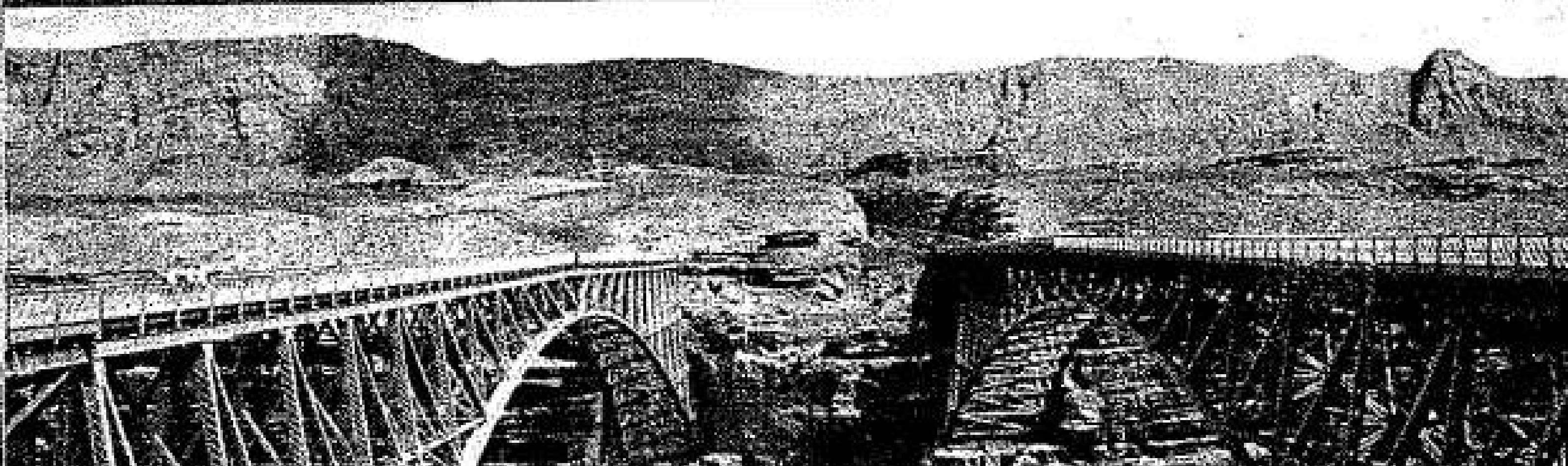
FOURTH EDITION

A TEXTBOOK OF

STRENGTH OF MATERIALS



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6.12. SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR A SIMPLY SUPPORTED BEAM CARRYING A UNIFORMLY DISTRIBUTED LOAD

Fig. 6.27 shows a beam AB of length L simply supported at the ends A and B and carrying a uniformly distributed load of w per unit length over the entire length. The reactions at the supports will be equal and their magnitude will be half the total load on the entire length.

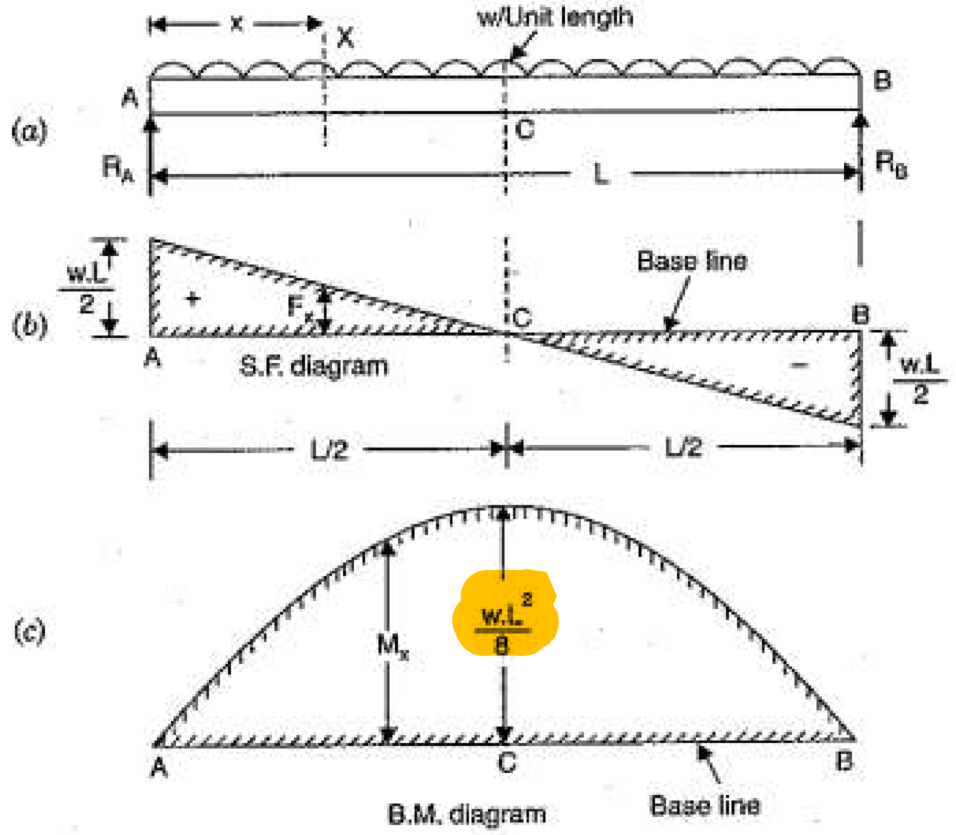

Fig. 6.27

Let R_A = Reaction at A, and

R_B = Reaction at B

$\therefore$ $R_A = R_B = \frac{w.L}{2}$

Consider any section X at a distance x from the *left end* A. The shear force at the section (*i.e.*, F_x) is given by,

$$F_x = +R_A - w\,.\,x = +\frac{w.L}{2} - w\,.\,x \quad \text{...(i)}$$

From equation (*i*), it is clear that the shear force varies according to straight line law. The values of shear force at different points are :

At A, $x = 0$ hence $F_A = +\frac{w.L}{2} - \frac{w.0}{2} = +\frac{w.L}{2}$

At B, $x = L$ hence $F_B = +\frac{w.L}{2} - w.L = -\frac{w.L}{2}$

At C, $x = \frac{L}{2}$ hence $F_C = +\frac{w.L}{2} - w.\frac{L}{2} = 0$

The shear force diagram is drawn as shown in Fig. 6.27 (*b*).

The bending moment at the section X at a distance x from left end A is given by,

$$M_x = +R_A\,.\,x - w\,.\,x\,.\,\frac{x}{2}$$

$$= \frac{w.L}{2}\,.\,x - \frac{w.x^2}{2} \qquad \left(\because\ R_A = \frac{w.L}{2}\right) \quad \text{...(ii)}$$

From equation (*ii*), it is clear that B.M. varies according to parabolic law.

The values of B.M. at different points are :

At A, $x = 0$ hence $\quad M_A = \frac{w.L}{2}.0 - \frac{w.0}{2} = 0$

At B, $x = L$ hence $\quad M_B = \frac{w.L}{2}.L - \frac{w}{2}.L^2 = 0$

At C, $x = \frac{L}{2}$ hence $\quad M_C = \frac{w.L}{2}.\frac{L}{2} - \frac{w}{2}.\left(\frac{L}{2}\right)^2 = \frac{w.L^2}{4} - \frac{w.L^2}{8} = +\frac{w.L^2}{8}$.

Thus the B.M. increases according to parabolic law from zero at A to $+\frac{w.L^2}{8}$ at the middle point of the beam and from this value the B.M. decreases to zero at B according to the parabolic law.